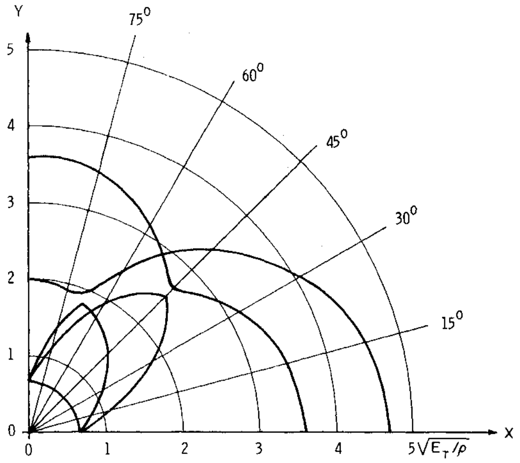

Fig. 3 Wave velocity surfaces, $c(\theta)$, in the first quadrant of the (0°/90°/90°/0°) plate.

sidered. The material properties of these layers are described by the following engineering constants‡:

$$E_L = 25 \times 10^6 \text{ psi}, \quad E_T = 10^6 \text{ psi}, \quad G_{LT} = 0.5 \times 10^6 \text{ psi}$$
$$\nu_{LT} = 0.25, \quad \nu_{TT} = 0.35, \quad \rho_0 = 0.073 \text{ lb/in.}^3$$

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and the subscripts L and T indicate directions parallel and normal to the fibers, respectively.

We have considered two plates each of which is made of four layers having different lay-up angles and/or lamination sequences, namely: a) (0°/90°/0°/90°); b) (0°/90°/90°/0°), where the angles are positive when measured counterclockwise from the x axis to the fiber direction.

The roots of the vanishing determinant of Eqs. (8) are determined by the Newton-Raphson technique. In general, five distinct roots representing five possible wave speeds exist in any given direction. However, in certain preferred directions, the material properties are such that repeated roots may exist.

Figures 2 and 3 show the wave velocity surfaces in the first quadrant of the x, y plane, for (0°/90°/0°/90°), and (0°/90°/90°/0°) laminates, respectively. The velocity is nondimensionalized by a factor of $(E_T/\rho_0)^{1/2}$. It is pointed out that the slowest velocity surface, which is associated with the transverse displacement w, is uncoupled from the other four surfaces, since the material is monoclinic. However, all the other four velocity surfaces may be severely coupled. On each of the corresponding wave surfaces, discontinuities in normal forces, shear forces and bending and twisting moments exist simultaneously. Their relative magnitudes may be determined from Eqs. (8) once the wave speeds c have been determined.

Multiple coupled one-dimensional stress waves in a heterogeneous plate were first treated by Wang, Chou, and Rose[6] using the method of characteristics. Recent experiments conducted at Drexel Univ. using a low-frequency ultrasonic transducer to produce normal-to-the-plate pulses show only two distinct wave groups traveling in the plane of the plate. Clearly, a definitive conclusion cannot be drawn at least presently, due to the limited data available.

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‡ These values are typical of high modulus graphite-epoxy composites. Such material layers may be considered as being square symmetric. The computation for the plates' rigidities and their transformation to local coordinates was carried out following the outlines in Refs. 4 and 5.

Three-Dimensional Separation for Interaction of Shock Waves with Turbulent Boundary Layers

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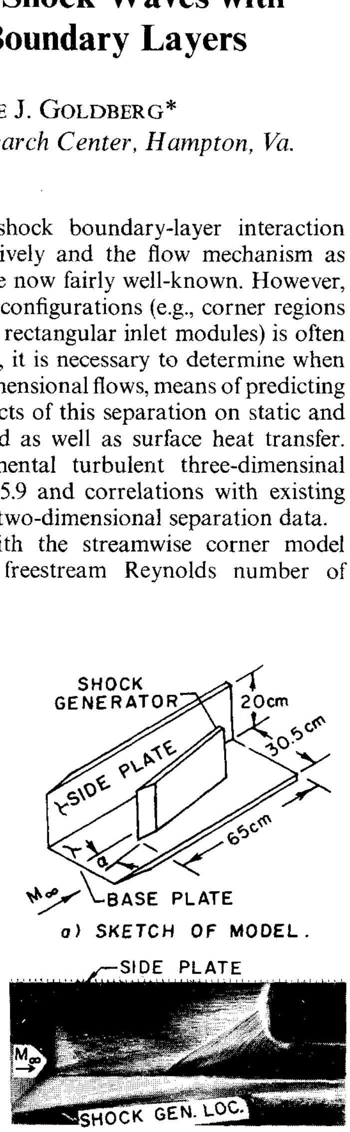
TWO-DIMENSIONAL shock boundary-layer interaction has been studied extensively and the flow mechanism as well as separation criteria are now fairly well-known. However, flow separation on practical configurations (e.g., corner regions such as wing-body junctions, rectangular inlet modules) is often three-dimensional. Therefore, it is necessary to determine when separation occurs in three-dimensional flows, means of predicting such separation, and the effects of this separation on static and total pressure in the flowfield as well as surface heat transfer. This Note presents experimental turbulent three-dimensinal separation results at $M_\infty = 5.9$ and correlations with existing three-dimensional as well as two-dimensional separation data.

This study was made with the streamwise corner model sketched in Fig. 1a at a freestream Reynolds number of

Fig. 1 Streamwise corner model.



b) OIL FLOW, $\alpha = 16°$, $\delta_b = 5.0$ cm.

Received April 20, 1973; revision received June 18, 1973.
Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow; Boundary Layers and Convective Heat Transfer—Turbulent.

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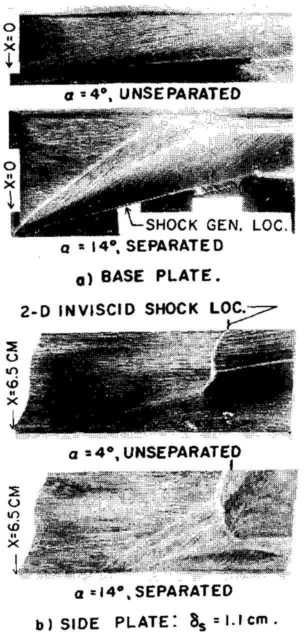

Fig. 2 Oil flow patterns. $\delta_b = 5.0$ cm.

2.4×10^7/m in the Langley 20-Inch Mach 6 Tunnel. The base plate boundary-layer thickness at the shock generator leading edge (δ_b) was 0.45 and 5.0 cm when mounted in the freestream and flush with the tunnel floor, respectively. The side plate boundary-layer thickness (δ_s) at the inviscid incident-shock location was 1.1 cm for all tests. (The shock generator was located at the same axial station but was moved away from the side plate with increasing α such that the inviscid incident shocks would intersect the side plate at the same axial station). An oil flow photograph for the shock generator at 16° angle of attack is shown in Fig. 1b (the shock generator was removed after each test for photo to show both surfaces), and represents the type of flow patterns used to determine incipient separation.

Typical examples of unseparated and separated oil flow patterns resulting from shock boundary-layer interactions with the base plate and side plate are presented in Fig. 2 (photos were taken normal to surfaces). A comprehensive review of the many possible three-dimensional separation surface patterns has been given by Wang.[1] Separation on the base plate results from the base plate boundary layer interacting with a glancing shock (herein defined as a shock with the flow deflections in a plane parallel to the surface under consideration). This separation is characterized by the upstream oil lines turning sharply and forming a thin, nearly conical oil accumulation line as seen in Fig. 2a. Part of the flow merging into this oil line apparently lifts off the surface; otherwise the accumulation line would thicken as the adjacent streamlines meet at the interaction. For the unseparated case the oil lines do not coalesce but simply turn away from the shock generator. The glancing-shock separation shown herein for turbulent flow appears to be very similar to the "free vortex layer" that was earlier proposed for laminar flows.[1,2]

Separation on the side plate (Fig. 2b) is characterized by two different surface flow patterns. The first pattern is the thin oil accumulation line which originates at the intersection of the base plate separation with the side plate. (Note that Figs. 2a and 2b are not aligned; however, this intersection is clearly seen in Figure 1b). This glancing-shock separation is produced by the separation on the base plate interacting with the side plate boundary layer. The second separation pattern is seen downstream of the nearly vertical oil accumulation line which forms near the two-dimensional inviscid incident-reflected shock location; (an incident-reflected shock is herein defined as a shock with the flow deflection in a plane normal to the surface under consideration). This separation behind the incident-shock interaction is characterized in the surface oil flow patterns by a strong outflow which originates near the base of the vertical oil accumulation line; placing a fence at the edge of the side plate verified that the strong outflow is not due to end effects. Separation in this region is herein referred to as incident-shock separation; however, it is caused by the effects of both the three-dimensional flow resulting from the glancing-shock interaction and the incident-reflected shock interaction. This separation occurs at a lower shock generator angle than that required for the glancing-shock incipient separation. The more complex incident-shock separation does not seem to correspond to any previous single separation criteria but is apparently a mixture of Maskell's[2] "free-vortex layer" and "bubble" type.

A partial compilation of incipient separation angles for two-dimensional turbulent flow as a function of local Reynolds number based on boundary-layer thickness (R_δ) from Coleman[3] and Holden[4] is shown in Fig. 3. Incipient separation angle is the maximum wedge angle, or equivalent flow turning angle for incident-reflected shocks, at which separation first appears: for glancing-shock interaction, it is the shock generator angle; for incident-reflected shock, it is twice the shock generator angle. Although all the two-dimensional data are not in perfect agreement, they do permit the selection of realistic angles to avoid incipient separation. Included in Fig. 3 are the maximum equivalent flow turning angles obtained in the present three-dimensional investigation at $M_\infty = 5.9$ for incident-reflected shocks with the side plate. The present three-dimensional data are much below the two-dimensional data; the cause for this is apparently associated with the glancing-shock interaction on the base plate which affects a significant portion of the side plate boundary layer (see Fig. 1b). Therefore, the glancing-shock interaction will first be discussed as it appears to govern all the separation in the present experiments.

Across the glancing shock, only the normal component of the upstream Mach number undergoes a velocity change. Therefore, the component of the upstream Mach number normal to the inviscid shock (M_n) for the three-dimensional incipient glancing-shock separation angles (shock generator angle) from $1.6 < M_\infty < 5.9$ are presented in Fig. 3 (open symbols). The three-dimensional data presented this way follow the trend of the two-dimensional data. Thus, at least for a first approximation, the three-dimensional separation angles for incipient separation with glancing-shock interactions can be determined from existing two-dimensional data. Further analysis using all available data indicates that this approximation is better at supersonic than at hypersonic Mach numbers. However, there is only a small difference in the three-dimensional separation angles for $1.6 < M_\infty < 5.9$ and a fairing of the data is probably sufficient for most uses. This is particularly true since the present $M_\infty = 5.9$ data show almost no effect of Reynolds number from 10^5 to 10^6.

The incident-shock separation on the side plate, Fig. 3, shows that the three-dimensional separation angles (solid symbols)

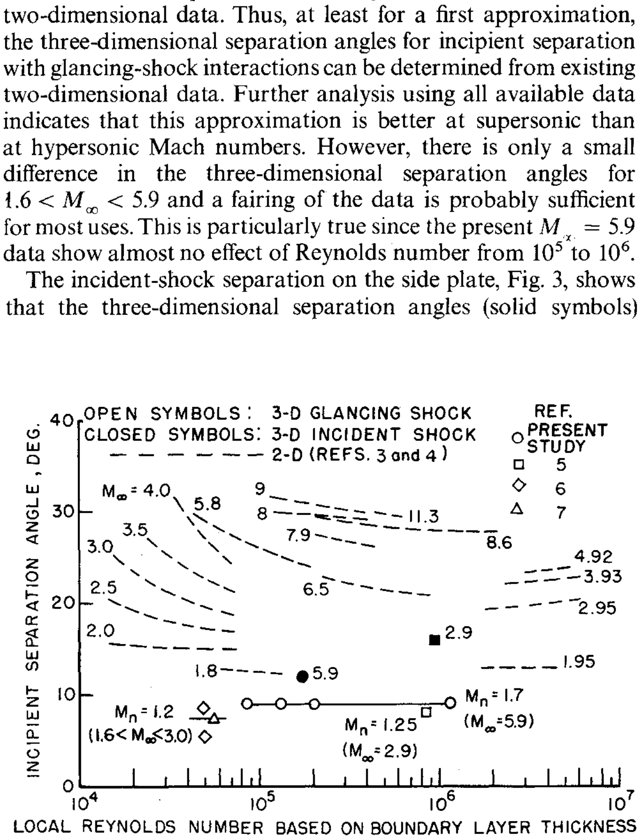

Fig. 3 Incipient separation for turbulent flow.

decrease with increasing freestream Mach number, which is contrary to results for glancing-shock and two-dimensional separation. Also, the three-dimensional separation angle from the incident-shock interaction is approximately half the two-dimensional value at $M_\infty = 5.9$ but nearly equal to the two-dimensional value at $M_\infty = 2.9$. A probable explanation of this is that the large lambda shock formation which occurred in the present experiments did not occur for Reda and Murphy[5] at $M_\infty = 2.9$. (Compare oil flows in Fig. 6 of Ref. 5 with Fig. 2 of this Note). This lambda shock results from flow disturbances on the base plate which extend considerably further upstream of the inviscid glancing-shock location for hypersonic flow than for supersonic flow as illustrated by the oil flows in Fig. 8 of Ref. 5 and Fig. 2 of this Note. From a practical standpoint, this incident-shock interaction result indicates that separation will occur in hypersonic rectangular inlets at smaller flow deflection angles than in supersonic inlets provided that other conditions are similar. This is an unexpected result with important implications.

In summary, it has been shown that separation occurs much earlier for turbulent three-dimensional flow than for two-dimensional flow at hypersonic speeds. Thus, for conditions when both types of separation can occur (such as in hypersonic inlets) three-dimensional rather than two-dimensional separation criteria should be used. A first approximation of three-dimensional incipient separation angles, which are apparently insensitive to turbulent Reynolds numbers, has been given.

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An Attempted Improvement of the Method of Integral Relations for a Blunt Body in a Supersonic Flow

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THE problem of calculating the flow in the shock layer ahead of a blunt-nosed body in a supersonic stream has been the subject of many investigations. In recent years, two methods have been developed which have received widespread use: these are the time-dependent method, the most effective form for blunt-body work being that due to Moretti,[1] and the Method of Integral Relations, which was developed by Belotserkovskii and co-workers.[2,3]

In the Method of Integral Relations, the partial differential equations of motion are replaced by a set of simultaneous, first-order ordinary differential equations. The number of equations depends on the degree of approximation being attempted. The numerical solution of this set as an initial value problem is not difficult, but a complication arises in that the initial values of some of the dependent variables are not known a priori and must be found by applying regularity conditions at certain singular points of the solution. In the first approximation, one initial value (the initial shock displacement) is not known and one regularity condition must be satisfied. It is found that a high degree of accuracy in the initial shock displacement is needed before an acceptable solution is obtained, but nonetheless, the computing time and programing labor involved are by no means excessive. Higher approximations, involving the application of more regularity conditions, have been used by Belotserkovskii and his associates, but it appears that the computing time and effort required in applying the regularity conditions become large so that, even in the second approximation, the method becomes rather unattractive.

Because of these difficulties, Traugott[4] suggested some years ago a modification which would improve the degree of approximation without introducing any further regularity conditions. Traugott's suggestion was as follows. In the first approximation of the Method of Integral Relations, the governing equations are integrated across the shock layer and the variations of certain flow functions across the layer are approximated linearly. In the nth approximation, the shock layer is divided into n strips and integrations are performed to each strip boundary; the flow function variations are now approximated by an nth order polynomial and, it turns out, there are n regularity conditions to be applied. Traugott pointed out that even if the shock layer is not subdivided, the flow variations can be approximated by cubics if use is made of expressions for the vorticity at the body and at the shock. This improved approximation is achieved without introducing further regularity conditions. The initial formulation of the equations is a little involved but, once accomplished, the computation would be expected to be only slightly longer than the conventional, linear calculation. In his original paper, Traugott used cubics to find corrections to the linear solution, not altogether successfully. Surprisingly, no further work on this approach appears to have been reported.

Some improvement over the first approximation is particularly desirable for the cases of low Mach number flows over bodies with small curvature noses[4] and of low Mach number supersonic jets impinging on flat surfaces.[5] It was therefore decided to formulate and program Traugott's suggestion for the case of a flat-faced, axisymmetric body. Numerical difficulties were encountered on running the program. Very considerable effort was put into checking the formulation and coding: these were found to be correct. The source of the difficulty was finally traced to an inherent instability[6] in the governing differential equations. No way around this difficulty can be seen and it is concluded that Traugott's suggestion, while physically attractive, does not lead to a viable method of calculation, at least in this case. Even in retrospect, there does not seem to be any way in which the instability could have been foreseen.

The purpose of this very brief Note is to prevent others from embarking on the considerable labor of following up Traugott's suggestion, only to arrive at the disappointing result reported here. A report giving a complete formulation of the equations and a discussion of the instability is available from the author.[7]

Received April 20, 1973; revision received June 29, 1973. Helpful comments on the numerical aspects of the problems were made by J. F. Baldwin and J. H. Sims Williams and are gratefully acknowledged.

Index category: Supersonic and Hypersonic Flow.

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